Texture-Induced Modulations of Friction Force: The Fingerprint Effect

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(Received 13 July 2011; published 11 October 2011)

Modulations of the friction force in dry solid friction are usually attributed to macroscopic stick-slip instabilities. Here we show that a distinct, quasistatic mechanism can also lead to nearly periodic force oscillations during sliding contact between an elastomer patterned with parallel grooves, and abraded glass slides. The dominant oscillation frequency is set by the ratio between the sliding velocity and the grooves period. A model is derived which quantitatively captures the dependence of the force modulations amplitude with the normal load, the grooves period, and the slides roughness characteristics. The model’s main ingredient is the nonlinearity of the friction law. Since such nonlinearity is ubiquitous for soft solids, this “fingerprint effect” should be relevant to a large class of frictional configurations and have important consequences in human digital touch.

DOI: 10.1103/PhysRevLett.107.164301 PACS numbers: 46.55.+d, 68.35.Ct, 81.40.Pq
Typical $F_S$ signals in steady sliding are shown on Figs. 2(a)–2(d) as a function of the scanned distance $u = u,t$ with $v = 0.1 \text{ mm s}^{-1}$ perpendicular to the direction of the ridges and $F_N = 0.5 \text{ N}$. Minute but clearly measurable nearly periodic oscillations are observed, with no equivalent in the normal force signals. Their amplitude weakly depends on $\lambda$ and increases with $F_N$. They are significantly more pronounced with the rough+ than with the rough- substrate [Figs. 2(b) and 2(d)]. The effect entirely vanishes when the direction of the ridges is aligned with the direction of motion. In the range of velocities explored, these modulations are independent of $v$. All experiments presented further are thus done at $v = 0.1 \text{ mm s}^{-1}$ and $u$ is taken as the time-varying variable.

The power spectrum $S(\delta F_S)(q)$ of $\delta F_S(u) = F_S(u) - \bar{F}_S$ (the bar stands for time averaging) was computed by averaging over a 20 mm travel distance. All spectra exhibit well-defined peaks at $q = 2\pi/\lambda$ at corresponding harmonics [Fig. 2(e)]. $S(q) \propto A^q$; at $q = q_1$ increases with $F_N$ as $AF^q_N$ (Fig. 3) where values of $A$ and $q_1$ are collected in Table I. As mentioned above, $A$ depends weakly on $\lambda$ and is significantly higher for the rough+ than for the rough- substrate. The exponent $\nu$ is close to 1 for all experimental configurations.

In parallel to the force measurements, rapid imaging of the contact zone was performed using a fast camera (Fastcam APX-RS, Photron, Japan) operating at 60 Hz. At all loads, contact occurs only at the summits of the ridges, thus yielding a large contrast between the top and bottom of the pattern and allowing for a precise tracking of the edges of the ridges with $\approx 10 \mu\text{m}$ accuracy. No stick-slip motion of the ridges was observed. Thus, the measured force oscillations cannot be accounted for by periodic stick-slip events whose frequency is set by the period of the pattern as reported in [2]. This is further supported by the observation that the fluctuations are quasistatic ($\nu$ independent).

In the rest of this Letter, we aim at understanding the physical origin of the observed friction force modulation and the dependence of its amplitude with the normal load, pattern period, and substrate roughness characteristics. We first consider a perfectly smooth elastomer surface (no patterning) rubbed against an abraded glass slide. We denote $p(x,y,u)$ and $\tau(x,y,u)$ the normal and tangential stress field at a given position $u$ of the substrate, such that $F_N(u) = \int p(x,y,u)dx\,dy$ and $F_S(u) = \int \tau(x,y,u)dx\,dy$. Under a local Amontons-Coulomb friction law assumption, i.e., $\tau(x,y,u) = \mu\,p(x,y,u)$ with $\mu$ a uniform (material dependent) friction coefficient, the friction force reads $F_S(u) = \mu_0F_N$ and is thus expected to be time independent. We therefore hypothesize a weakly nonlinear relationship between $p(x,y,u)$ and $\tau(x,y,u)$ which is equivalent to postulating a pressure dependence of the local friction coefficient, i.e., $\mu = \tau/p = \mu(p)$. The friction force now reads

$$F_S(u) = \int \mu(p)p(x,y,u)dx\,dy. \quad (1)$$

The spatial variations of $p(x,y,u)$ can be decomposed into a time-averaged component $\bar{p}(x,y)$ set by the macroscale...
plane-on-sphere contact geometry and a time-fluctuating component $\delta p(x, y, u)$ associated with the microscopic roughness of the glass slide. For simplicity, we further assume that the elastomer is in intimate contact with the glass surface. This assumption is expected to fail at the microasperities scale but should be valid at intermediate scale such as the interridge distance $\lambda$. In this limit, the texture-induced pressure modulations are set by the topography of the glass substrate and are independent of the microasperities scale but should be valid at intermediate scale. This condition imposes that the stress at the surface of the elastomer imposes that the stress associated with the microscopic roughness of the glass slide. For simplicity, we further assume that the elastomer is in intimate contact with the glass surface. This assumption is expected to fail at the microasperities scale but should be valid at intermediate scale such as the interridge distance $\lambda$. In this limit, the texture-induced pressure modulations are set by the topography of the glass substrate and are independent of the microasperities scale but should be valid at intermediate scale.

The normalization factor $K(u)$ ensures that $F_N$ remains constant during sliding. This condition imposes

$$K(u) = \left(1 + \frac{1}{F_N} \int \delta p(u - x, y) dxdy \right)^{-1}$$

The latter expansion is valid when the contact diameter is much larger than the texture scale. Indeed, in this limit, the integral of the roughness-induced pressure field becomes vanishingly small with respect to the confining force $F_N$. Rewriting Eq. (1), one obtains

$$\delta F_s(u) = F_s(u) - \bar{F}_s = \int [\mu(\bar{p}) - \langle \mu \rangle] \delta p(u - x, y) dxdy,$$

where $\langle \mu \rangle$ is defined as the ratio $\bar{F}_S/F_N$.

The friction force fluctuations thus appear as the convolution product of a function characterizing the friction coefficient spatial heterogeneities and the texture-induced pressure modulations field. The presence of regular ridges at the surface of the elastomer imposes that the stress between ridges vanishes. This can be accounted for, in first approximation, by introducing a $\lambda$-periodic Heaviside function $H(x) = \theta[\sin(2\pi x/\lambda)]$ under the integral, which directly results in a spectral selection of the associated spatial mode.

In Fourier space $[13]$, the fluctuating component of the friction force thus reads

$$S_{\delta F_s}(u) = (2\pi)^4 \int dq_u \left| F(H(x)[\mu(x, y) - \langle \mu \rangle]) \right|^2 S_{\delta p}(u).$$

Wave vector $q_u$ ($q_v$) refers to the parallel (perpendicular) direction to the sliding direction.

Under the assumption of intimate contact, $S_{\delta p}(u)$ can be simply expressed as a function of the roughness spectrum $C(q)$ as $S_{\delta p}(q) = (E^*/2)^q^2 C(q)$, where $q$ is the norm of $(q_u, q_v)$ and $E^*$ the reduced Young's modulus. Lacking any established constitutive law for our system, the friction coefficient spatial field, on the other hand, is estimated empirically from global force measurements. For all experimental configurations, a power-law dependence is observed between both force components $\bar{F}_S = B F_N^\gamma$ with $\gamma = 0.87 \pm 0.04$ (Fig. 4). We postulate a power-law relationship between the local shear stress and the local pressure $\tau(x, y) = \beta \bar{p}(x, y)$ [15]. Assuming that $\bar{p}(x, y)$ follows a Hertz profile, $\bar{F}_S$ can be derived by analytically integrating $\tau(x, y)$ over the contact area, yielding $\gamma = (m + 2)/3$ and an exact relationship between $\beta$, $\bar{B}$, and $\gamma$. Figure 4 shows a comparison between $\bar{F}_S$ versus $F_N$ obtained experimentally and through integration of $\tau$ over the contact area. Note that the choice of a power law is simply dictated by the fact that it constitutes the simplest functional form consistent with the global force measurements in the explored range.

Both predicted and measured amplitudes of $S_{\delta F_s}(u)$ with $F_N$ are presented on Fig. 3, for all experimental
The amplitude of these force fluctuations can then become an order of magnitude larger than in frequency. The mode selection of the force fluctuations at a particular texture-induced pressure modulation which induces a surface of the block operates a spectral filtering of the scale. In contrast, the presence of a regular pattern at the substrate (here the PDMS block) is smooth and the contact is expected to be in practice hardly detectable when the fixed force fluctuations should occur. However, these are the local and global friction coefficients are not the same slightly decreases with \( C_2 \), and falls rapidly to zero when \( h \) approaches unity, i.e., when Amontons-Coulomb friction's law is assumed locally.

This study shows that any nonlinearity in the friction law leads to the development of texture-induced friction force fluctuations. Since purely linear friction laws are scarcely observed, the present mechanism should be relevant to most practical situations. More generally, whenever the local and global friction coefficients are not the same [16] force fluctuations should occur. However, these are expected to be in practice hardly detectable when the fixed substrate (here the PDMS block) is smooth and the contact zone diameter is much larger than the typical roughness scale. In contrast, the presence of a regular pattern at the surface of the block operates a spectral filtering of the texture-induced pressure modulation which induces a mode selection of the force fluctuations at a particular frequency. The amplitude of these force fluctuations can then become an order of magnitude larger than in the smooth case as shown by the present study. This mechanism was here examined for the simplest possible pattern, i.e., regular parallel stripes, but richer force signal spectra should be similarly obtained with more complex patterns.

In essence, the macroscopic friction force signal carries information about the spectral content of the microscopic substrate topography at frequencies defined by the elastomer micropattern. A practical consequence is that \( F_s \) fluctuations amplitudes could be used to discriminate surfaces having small differences in their roughness at scales much smaller than the contact extension. Hence, under the same conditions, force fluctuations are roughly 10 times larger for the substrate having a \( h_{rms} = 5 \, \mu m \) than for the one with \( h_{rms} = 2.3 \, \mu m \). This mechanism could also facilitate the detection of motion. The transition between static and sliding contact should manifest as the sudden appearance of large single mode oscillations of the friction force. One may finally foresee the possibility to evaluate the scanning velocity, based on the sole friction force signal, by continuously extracting the peak frequency of these oscillations. We believe that this “spectrottribometry” approach may be relevant to human digital touch and prehension, owing to the presence of fingerprints, and that it could be easily implemented in tactile robotic sensing devices.

The authors deeply thank F. Petrellis for illuminating mathematical surgery, F. Zalamea for his valuable help, A. Chateauminois and C. Frétilny for fruitful discussions, and acknowledge financial support from ANR-DYNALO NT09-499845.

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